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Performance and Slip Factor Prediction for Radial and Mixed Flow Pumps

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Abstract— In the present paper, the slip phenomenon is studied numerically and experimentally for radial and mixed flow pumps. Sixteen cases are studied for pump with specific speed range from 10 to 75 with step 5; the slip factor is calculated for each case by using a new technique. The present calculated slip factor is validated with experiments and other results from literature. Experimental results were presented for radial and mixed flow pumps with specific speed values 29.3, 41, and 46.27. The effect of rotational speed is considered for case with specific speed 41 to investigate its effect on the slip factor prediction method. It is observed that the pump performance from experimental results is in a good agreement with the numerical one for all studied cases. However, the slip factor results show a good agreement only for radial pump cases with results from existing theoretical methods. The present new technique gives reasonable slip factor values for radial and mixed flow pump cases, which old theoretical methods have no reasonable for mixed flow cases. The present results show the weakness of the existing methods in predicting the slip factor for mixed flow pumps. A new mathematical formula is presented for calculating the slip factor including higher range of radial and mixed flow pumps.

Index Terms— Performance, Slip Factor, Prediction, Radial, Mixed, Pumps.

1 Introduction

Pumps are used in industrial facilities [1] to transfer fluids for processing, provide cooling and lubrication services and as a motive force provider in hydraulic systems. In the manufacturing sector, pumps represent 27 percent of motor electricity [2]. Centrifugal pumps have several applications such as water supply plants, irrigation, oil refineries, stream power plants, sewage, chemical plants, food processing factories, hydraulic power service. The aim of any pump is to transfer mechanical energy to the liquid flowing through it from some external source. Like any energy conversion process, the losses are occurred. Euler Equation is used to predict the energy transfer. There are losses occur in the energy transfer process between mechanical power of the impeller or runner and the fluid power. Many types of loss in centrifugal pumps can be classified to internal losses and external or mechanical losses. Hydraulic losses or blade losses is one of internal losses type, which changes of quantity direction occur at the sealing places between the housing and impeller at the rotary shaft seals. Sliding surface losses by bearing friction or seal friction are considered as external or mechanical loss is [3]. The common factor in the hydraulic loss is the flow slip which is not able to follow the exact blades angle. The actual exit flow angle β2 is smaller than the ideal exit flow angle of β_{2B} as shown figure 1. The phenomenon described quantitatively is determined by the "slip factor" or "angle of deviation" $\delta = \beta_{2B}-\beta_2$. Both terms implicitly assume the idea of identical blade flow and consider the real flow deviation outward from the angle of the blade outlet. The slip factor definition shows in equation 1

$$\sigma = \frac{c_{2u}}{c_{2u\infty}} \qquad (1)$$

The slip factor is defined as fluid slip in the impeller of a pump which is the deviation in the angle of the fluid leaves. In radial impellers slip is a very important phenomenon which is useful to determine the accurate estimation of input work or the energy transfer between the fluid and the impeller. The complicated problem for study slip for centrifugal pump impeller is how to predict the actual performance of moving fluid inside the impeller spam such as the actual velocity magnitude and the actual angle of fluid leaves the impeller [3]. Using CFD can solve this complicity as the flow characteristics can be predicted at any point of the pump model. So, in this paper, sixteen different specific speed centrifugal pumps are simulated using CFD and slip factor also will be predicted for all cases of centrifugal pump using a new technique. Experimental test rig also is constructed for a chosen three cases to validate the result of the simulated model.

Several methods have been introduced to predict the slip factor values which have a reasonable with the case of a redial flow pump type only and have a weakness with a mixed flow type such as, Stodola (1927) [4] obtained expression for the slip factor which considered the earliest and simplest one for determine the slip factor as equation 2.

$$\boldsymbol{\sigma = 1 - \frac{\frac{\pi}{Z}\cos\beta_{2B}}{1 - \emptyset\tan\beta_{2B}}} \qquad (2)$$

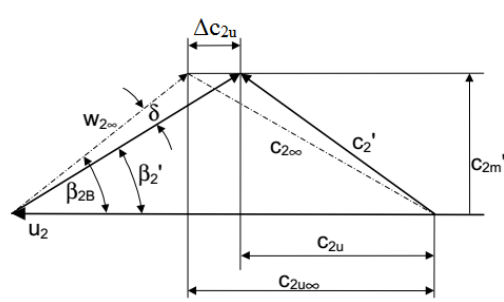


Fig 1 Slip and deviation angle

Using relaxation methods of calculation Stanitz (1952) [5] solve the potential flow field between the blades (blade-to-blade solution) for eight impellers which blade exit angles β2B varying from 0 to 45 deg. It is proposed that the computed slip velocity Δc_{2u} is independent of vane angle β_{2B} and depended only on blade spacing (number of blades). The corresponding slip factor is determined by equation 3.

$$\sigma = 1 - \frac{0.63\frac{\pi}{Z}}{1 - \emptyset\, tan\beta_{2B}} \qquad (3)$$

Wiesner (1967) [6] reviewed all the available methods for calculating values of slip factor and compared them with values obtained from tests.

$$\sigma = 1 - \frac{\sqrt{\cos\beta_{2B}}/Z^{0.7}}{(1 - \emptyset\tan\beta_{2B})} \qquad (4)$$

A simple analytical method to get the slip velocity from a Single Relative Eddy (SRE) introduced by Von Backström [7] 2006. In this study the other prediction methods are tried to unify. Because of none of the proposed methods are general and they produce different results even when it is applied to the same impeller. Experimental Investigation of slip Factors proposed by Mohamad Memardezfouli [8] in Centrifugal Pumps tested for theoretical and Experimental Models for the relationship between the effective slip Factor and the flow Rate in impellers and their Relation to velocity distribution.

H.A.Elsheshtawy [9] made numerical study of the slip factors in centrifugal pumps and study factors affecting their performance .Several factors that affect the value of the slip factor are found but most importantly are the angle of the exit blade which affect the direction of the slip factor and the impeller operation state. Yu-Liang Zhang [10] proposed that from the Euler equation basic of centrifugal impellers and experimental results of the external performance of the centrifugal pumps, a simple slip factor of centrifugal impellers calculation method is proposed Compared with Wiesner's original model.

A new slip factor model which appropriate with axial, radial, and mixed flow impellers proposed by Xuwen Qiu [11]. This model based on Stodola and Eck's slip factor modeling work as well as blade loading analysis.

S R Shah [12] got that CFD is the best tool which used in numerical simulation of centrifugal pumps for performance prediction at design and off-design conditions. Also, Reynolds average Navier-Stock (RANS) equations with two equation k-ε turbulence model are found to be appropriate to get a reasonable estimation of the general performance of the centrifugal pump, from an engineering point of view. The dereliction of use of the existing methods for estimating slip factor is that it depends on very complex measurements or use empirical formula, which is appropriate in certain cases. It is appropriate in the case of radial pumps and not suitable in mixed pumps and it does not have a clear effect in the case of the use of developers to perform such as adding Short or splitter blades. It is necessary to find another uncomplicated method for estimating the slip factor can be appropriate for all pump conditions.

The present technique is proposed as the ideal whirl velocity c2u∞ can be determined from drawing velocity triangle at the exit of impeller while the whirl in actual performance is difficult to be determined unless knowing the actual absolute velocity leaves the impeller W2. In present work , a complete model of pump is constructed and simulated by ANSYS 18.1 FLUENT program while the absolute velocity leaves the impeller can be observed and its value can be determined by area average integration of the velocities at the impeller exit plane [12] as a formula of equation 5.

$$W_2 = \int_0^{2\pi} W_{2\,CFD}\, d\theta \qquad (5)$$

2 Experimental Work Setup

The experimental work is used only for validate the numerical performances of pump models. The test rig used for investigating the studied cases is shown schematically in figure 2a and photographically in figure 2b. It consists of a closed cycle and is operated with water of ambient temperature. The open water store tank made of steel sizes maximally 1 cubic meter and the duration time for testing the pump is less than the duration time for change temperature, also density, of this amount of circulated water. The tested pump cases are with specific speed 29.3, 41, and 46.2. The reason of choosing these cases is to cover the two types of flow cases, radial and mixed, which pump with specific speed 29.3 is radial one and the other two pumps are mixed flow type. The magnetic flow meter with accuracy 0.01 m3/hr. is used for monitoring the discharge of the pump. Pressure at the pump outlet and inlet, which below atmospheric pressure, are monitoring by two calibrated gauge pressure values. The range of suction gauge pressure is from -1 to 1 bar and accuracy 0.05 bar while the discharge one with range from 0 to 7 bar and accuracy 0.1 bar. The main dimension of these pumps is shown in table 1.

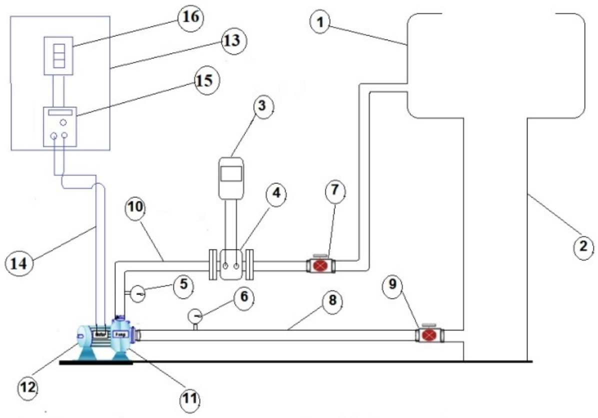
Table 1 Dimension for tested pumps

	Ns= 9.3	Ns= 41	Ns=46.2
Number of blades	6	6	6
shaft diameter mm	21.8	15.2	17.4
Suction diameter mm	117.6	100	90
Impeller diameter mm	235	165	135
Out width mm	20	20	20
Impeller blade thickness mm	3.5	2.8	2.3
Impeller out angel	15.3°	23.4°	21.2
Volute inlet diameter mm	240	185	150
Volute inlet width mm	33	31.3	30.7
Volute out diameter mm	95	95	70

3. Numerical Model

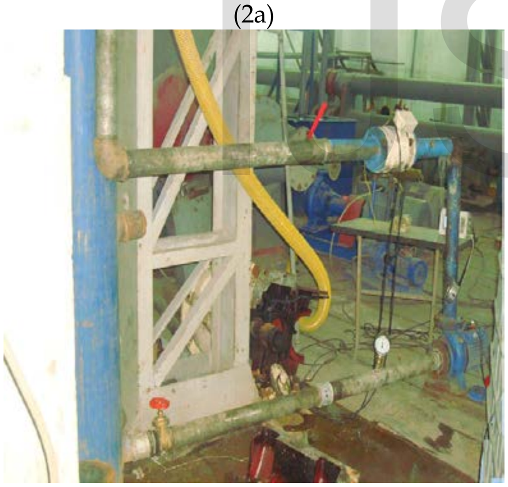
3.1 Computational model

A complete centrifugal pump 3D model is constructed using a turbo machine program. The suction and discharge pipe of the pump is designed with length equal fifth of their diameters to standardly monitoring the pressure value at the inlet, at the beginning of the suction pipe, and the outlet, at the end of discharge pipe. The impeller to be modeled is of single-entry closed type with hub and shroud face. The volute has circular cross section, and the design of shaping for impeller and volute is produced by a computer program shown in figure 3. Dimensions of all cases presented in table 2.



1	Open tank	9	3 inch gate valve
2	12 inch pipe	10	3 inch discharge pipe
3	Flowmeter display unit	11	Pump
4	3 inch magnitic flowmeter	12	Motor Drive
5	Discharge pressure gauge	13	Elictrical Board
6	Suction pressure gauge	14	Elictrical Wire
7	3 inch ball valve	15	Variable speed drive
8	3 inch suction pipe	16	Elictrical conductor

(2a)



(2b)

Fig 2 Experimental test rig (a) scheme diagrams and (b) Photograph of the test rig

3.2 Grid generation

Computational Fluids Dynamics (CFD) has already been used in engineering widely with the fast development of computer technology. One of the key ANSYS 18.1 FLUENT techniques is mesh generation technology which affects the simulation precision of numerical calculation directly. It's very important for the computational simulation of pump to refine the quality of mesh generation. Figure 4 shows sketch of the pump grid using automatic mesh modeler. Mesh program automatically chose the perfect type of mesh for each zone. the final number of nodes after process the mesh is 800000 nodes which by trying the model with 500000 nodes and more, the result doesn't change. And with element size value 0.6 mm.

TABLE 2 THE DESIGNING DIMENSIONS FOR ALL CASES OF PUMPS

N_S	Q M^3/HR.	H_M	N RPM	Z	D_{SH} MM	D_1 MM	D_2 MM	B` MM	B_{2B} °	D_3 MM
9.7	10.8	16.6	1450	6	14.5	68.8	229	10.76	25	241
15.7	30	17.1	1450	6	17.5	91	237	13.05	21.7	237
19.1	54	19.5	1450	6	22.3	108	256	17.4	23.7	276
26.2	108	20.4	1450	6	26.7	131	265	21.2	19.4	290
31.0	168	21.8	1450	6	32.3	149	281	25.9	23.8	311
35.3	192	20.1	1450	6	32.7	154	273	27.6	25	305
39.1	270	22	1450	6	31.3	170	289	31.2	21.3	330
46.9	300	18.5	1450	6	35.4	174	273	33.6	24.7	320
50.8	450	21.8	1450	6	44.1	198	299	38.9	27.2	360
56.0	360	16.5	1450	6	37.4	182	265	36.8	27.9	320
59.4	480	18.5	1450	6	41.2	199	287	42	22.6	350
65.5	390	14.1	1450	6	35.2	184	256	40.2	23.4	320
71	150	6.7	1450	6	20.2	133	180.5	30	23.8	227
76.2	132	5.6	1450	6	18.2	127	168	29.2	24.3	214

3.3 Boundary conditions

In the present study, the boundary conditions are used for inlet is mass flow and outflow for outlet. Outer walls were stationary, but the inner walls are rotational. There are interfaces boundary conditions between the stationary and rotational regions. And non-slip boundary conditions at the impeller blades and walls have been imposed, and the flow near the wall is determined using the slandered wall function.

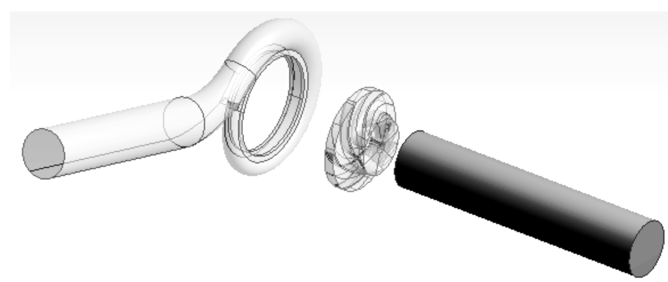


Fig 3 Numerical model CFD domain

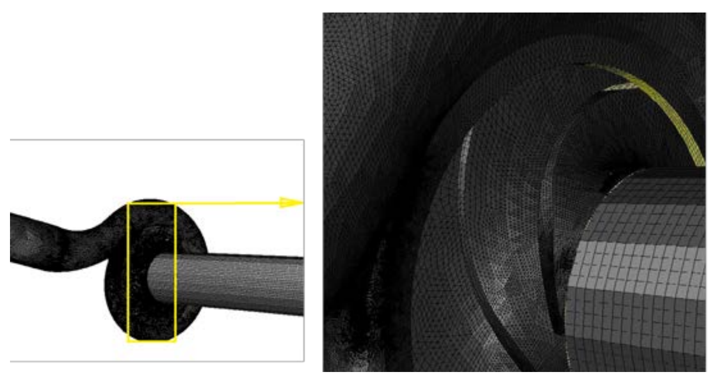


Fig 4 Three-dimension pump grid

3.4 Solver

Using ANSYS 18.1 FLUENT for simulate the inner flow field under non cavitation condition. The standard k-ε turbulence model and simple algorithm are applied to solve the equations of Navier-Stock [1] which Consider a three dimensional, incompressible flow with the relative velocities w_x, w_y, w_z in a Cartesian x,y,z coordinate system respectively ; the rotation is around the z-axis. For simplicity the continuity equation 6 and conservation of momentum 7 is written only for the x-direction according reference [3].

$$\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}+\frac{\partial w_z}{\partial z}=0$$

$$\frac{\partial w_x}{\partial t}+w_x\frac{\partial w_x}{\partial x}+w_y\frac{\partial w_x}{\partial y}+w_z\frac{\partial w_x}{\partial z}+\frac{1}{\rho}\frac{\partial \rho}{\partial x}-\omega^2 x+2\omega w_y=$$

$$v\left(\frac{\partial^2 w_x}{\partial x^2}+\frac{\partial^2 w_x}{\partial y^2}+\frac{\partial^2 w_x}{\partial z^2}\right)+\left(\frac{\partial \grave{\sigma}_x}{\partial x}+\frac{\partial \grave{\tau}_{xy}}{\partial y}+\frac{\partial \grave{\tau}_{xz}}{\partial z}\right)\frac{1}{\rho} \quad (7)$$

The simulation is not need to change with time because the performance of pump has to be studied in this case so the solution is steady and convergence precision of residuals in this case of simulation is 10 –5 according to previous works[13], [14], [15].

4. Results and discussion

4.1 Experimental and numerical results of cases Ns 29.3, 41, and 46.27

To verify the results from the simulation model, the numerical and experimental results comparison was done . the experimental and numerical performance of the pump with case of Ns 29.3 is shown in figure 5. The static pressure contours in figure 6 show the normally growing of the pressure through the impeller and volute fluid domain from the suction surface at the impeller to the exit surface in the volute, consequently it's easy to estimate the increasing of energy through the pump fluid domain by calculating the different of total energy at the water inlet and outlet surface which the total head factor of the pump are 1.11, 1.06 and 0.99 at flow factor 0.93, 1 and 1.12 respectively. Using Bernoulli equation to estimate the difference in total energy between suction face and discharge face, it's found that the values of total head factor measured in test rig which are 1.1, 1.06, and 0.98 at flow factor 0.93, 1 and 1.12 respectively which have a good match with the numerical one.

With the same method for estimating the performance experimentally and numerically figure 7 shows the experimental and numerical performance of case with specific speed 41while figure 8 shows the experimental and numerical performance of case 46.27 at different rotational speed which presents a good match between the numerical and experimental results also as case of specific speed 29.

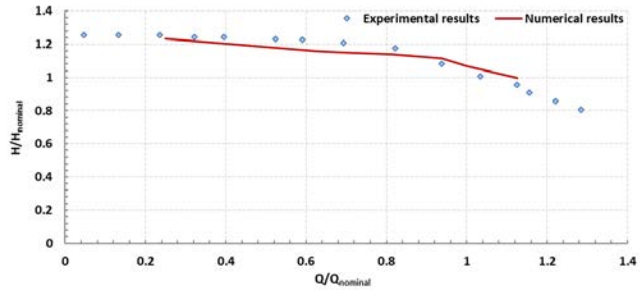
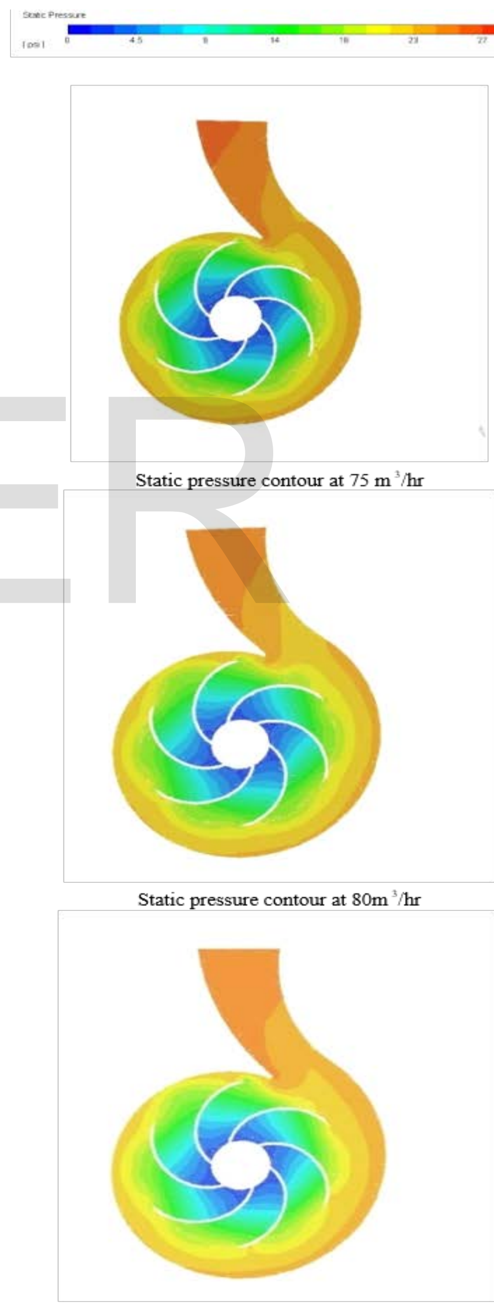


Fig 5 The experimental and numerical performance of pump with case Ns 29



Static pressure contour at 75 m ^{3}/hr

Static pressure contour at 80m ^{3}/hr

Static pressure contour at 90 m ^{3}/hr

Fig 6 Static pressure contour for case of Ns 29.3 in different flow rate

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The two figures below 9 and 10 shows the numerical result output from the CFD program for case of Ns 41 and 46.27 respectively. In the case of Ns 41 the pressure contours is presented at different flow rate 18, 43, 50, and 60 m^3/hr. which is compared with experimental result in figure 6 while figure 10 presents the pressure contours of case 46.27 at 2900 RPM at flow rate 52, 60, 70, and 76 m3/hr. and all numerical results at different rotational speed is presented in figure 8.

Also, to predict the slip factor for these cases must estimate the actual flow velocity from the numerical simulation with the same method of other cases. Figure 11 shows the absolute velocity profile for impeller exit plane for three cases Ns 29.3, 41, and 46 which the contours show the variation of the absolute velocity around the spam of blades. The actual absolute velocity can be determined by integrate the values of velocity leaves the impeller as shown in histogram of the values of absolute velocity leaves the impeller versus the possibility of occurrence for the three cases of Ns figure 12. Figure 13 shows the drawing actual and ideal velocity triangles for chosen cases of Ns 29.3, 41, 46, 9.7, 71.

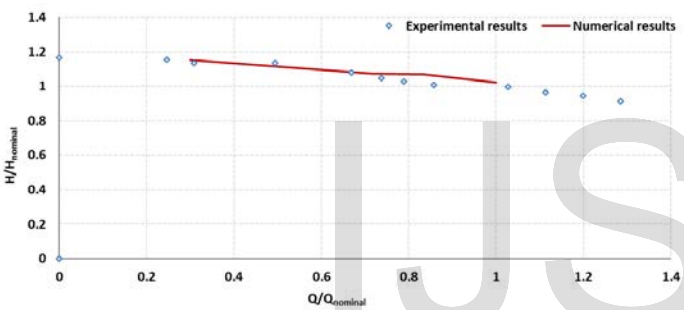

Fig 7 The experimental and numerical performance of pump with case Ns 41

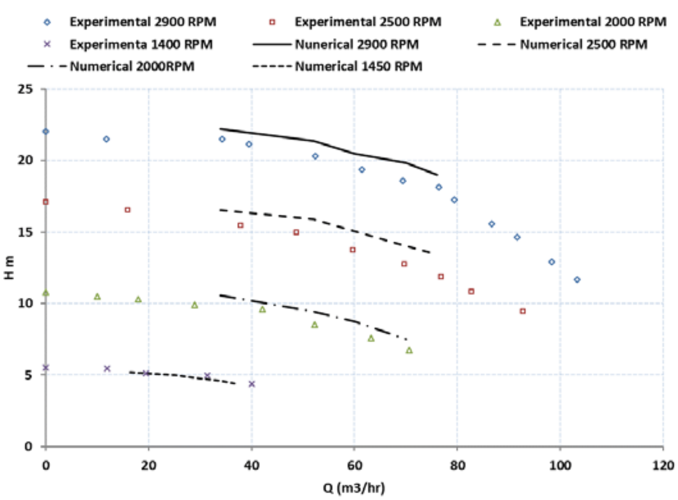

Fig 8 The experimental and numerical performance of pump with case Ns 46.27 at different RPM

The absolute velocity mostly equal the average value which is the most frequent value which is equal 10.5 m/s in the case of Ns equal 29.3 and the actual whirl velocity from the drawing actual triangle is 10.3 while the slip factor is equal 0.73.

4.2 Numerical Results

The values of slip factor estimated by the old method remain constant at different pump conditions which the specific speed, head, and discharge change. This is not logic because of the old methods depend on empirical formulas with some assumption make it far away from the actual performance of the centrifugal pump. Each case of centrifugal pump has special performance according velocities in and out from the impeller, the effect of vortex between impeller blades, and the fluid slipping at the outlet face of the impeller. All these effects only showed when the inner field of the impeller is analyzed using the simulation models. All parameters are estimated at any surface of the pump model. It’s easy to analyze the contour of the velocities leaves the impeller as shown in figure 11. The absolute velocity of the fluid varied through the impeller outlet face. The contour show that the maximum velocity of the fluid is near the moving walls or the impeller. And increase at the blade discharge side while decrease at the blade suction side in case of Ns = 29.3 which is radial flow impeller. This phenomena doesn’t occur in case of mixed flow impeller(Ns= 41 and 46) which the value of the absolute velocities at the blade discharge side are close to its value at the blade suction side because of the fluid in this case don't take the same blade angle plane when leaving the impeller. This explains that the slipping of fluid is decreased in case of mixed flow impeller.

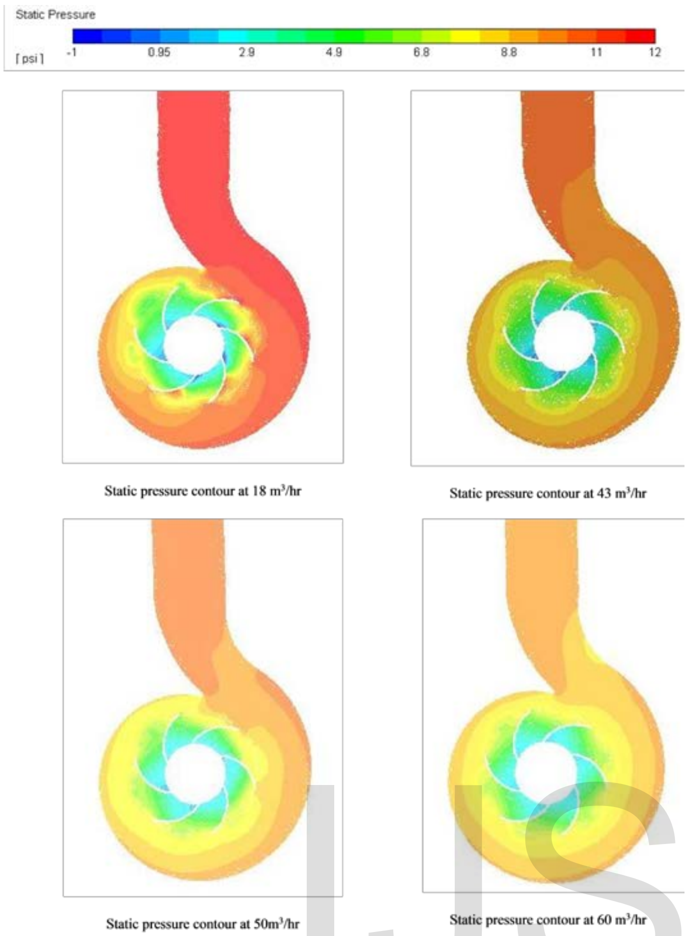

Fig 9 Static pressure contour for case of Ns 41 in different flow rate

The actual absolute velocity leaves the impeller W2 also is estimated as shown in table 3. The value of W2 is the average of the velocity values leave the impeller according the histogram shown in figure 12. This value is closer to the actual performance of the pump impeller. Now it's easy to draw the actual velocity triangle at outlet of impeller which is shown in figure 13.

TABLE 3 THE ACTUAL ABSOLUTE VELOCITY LEAVES THE IMPELLER FOR ALL CASES

N_s	9.7	15.7	19.1	26.2	31	35.3	39.1	46.9	50.8	56	59.4	65.3	71	76.2
W_2 m/s	12.5	10.6	11.7	11.5	11.7	11.5	11.2	10.6	11.1	10.2	11.25	9.8	7.2	6.9

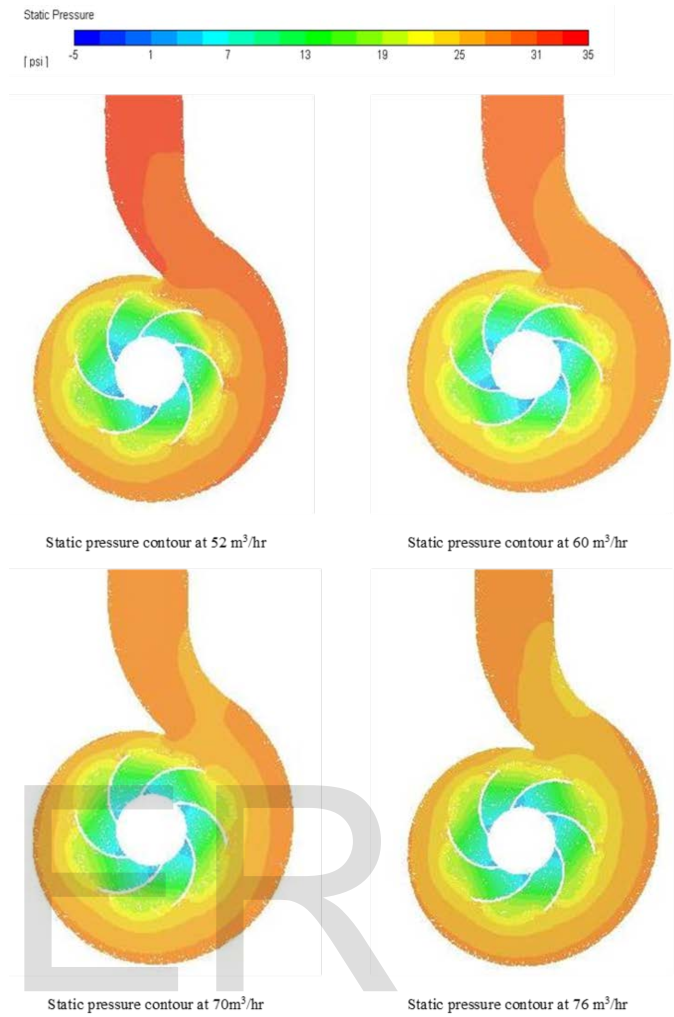

Fig 10 Static pressure contour for case of Ns 46.27 in different flow rate at 2900 RPM

It is clear from the results that when using the existing methods for calculation slip factor it’s found that the values of slip factor are not affected or slightly affected by the change in the values of specific speed because these methods depend on empirical formulas which does not show the effect of specific speed, especially when used in the case of mixed flow pump. However, when using the present method, it’s obviously clear that the slip factor change with varying the specific speed so that the slip factor will be affected by changing the values of pressure and the amount of water affecting the rotor of the pump and this is logical and known from the basic definitions of the slip factor.

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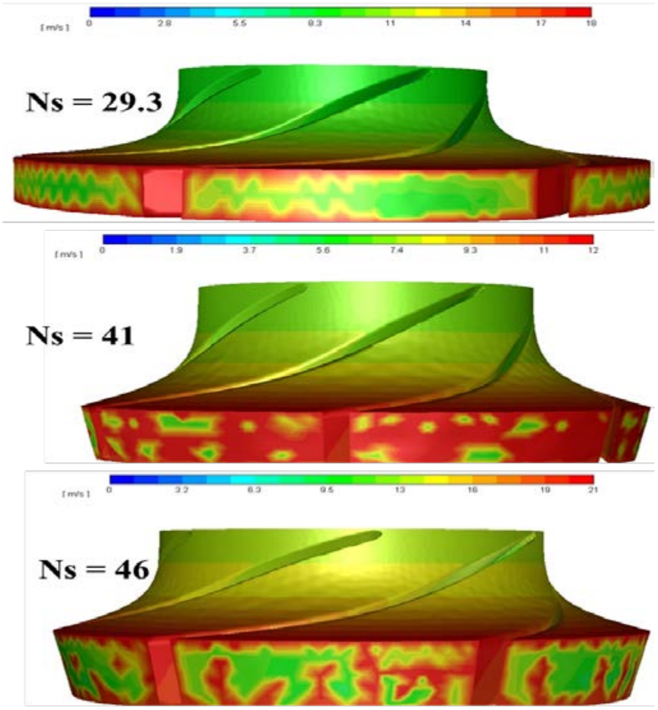


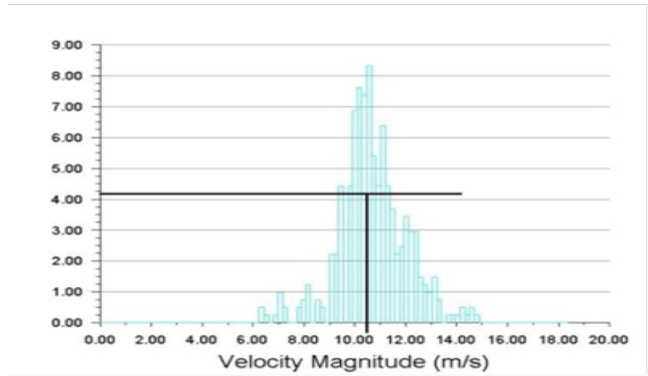
Fig 11 Absolute velocity contours

5. Relationship between Slip Factor and Specific Speed

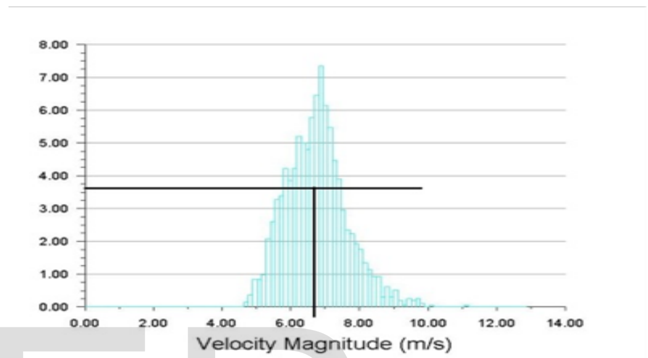
According the present results, when trying to apply relationship between the slip factor and specific speed only, it's found, when using linear or nonlinear regression program, that the values of residual for this relation is high. The relationship will be significant when adding the flow coefficient and the flow exit angle to the relation and firstly found a linear relationship with value of R2 equal 0.87 which presented in equation 8 and the residuals are shown in figure 14 but this value of R_2 is not good so a nonlinear relation is found which is a logarithmic second order of degree for all independent variables and the value of R2 of this relation is 0.94 and the residuals be small. Equation 9 shows the relation of the slip factor also Figure 15 shows the different of slip factor predicted and present also the values of residuals are shown.

$$\sigma = 0.94 + 0.013N_s - 3.68\Phi - 0.56\tan\beta_{2B} \qquad (8)$$

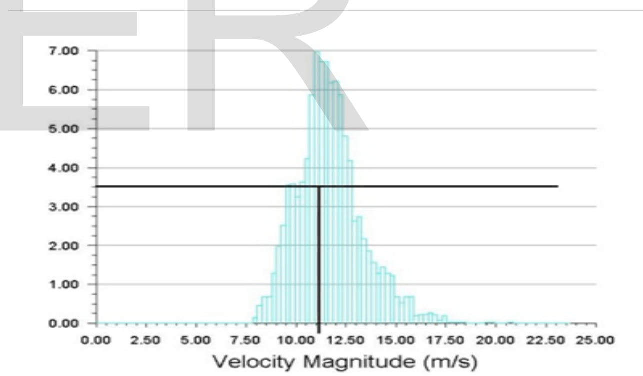
$$\sigma = -7.009 + 4.257 \log N_s - 2.915 \log\Phi + 1.97\log \tan\beta_{2B} - 0.5(\log N_s)^2 - 0.15(\log\Phi)^2 + 3.97(\log \tan\beta_{2B})^2 \qquad (9)$$



$N_s = 29.3$



$N_s = 41$



$N_s = 46$

Fig 12 Histogram of the values of velocity at the impeller exit

6.Conclusion

The numerical work of sixteen radial and mixed flow pumps have been presented in this paper aimed to predict the slip factor of each case using new technique. After calculating the real flow velocity leaves the pump impeller it's easy to calculate the real slip factor of any radial or mixed flow pump. The real and theoretical slip factor different are clearly shown especially in mixed flow pumps which in theoretical one there is small change in slip factor values with changing the specific speed, but this change is clear in present slip factor. The relationship of slip factor and specific speed is not affected unless adding the effect of the flow coefficient and the flow exit velocity.

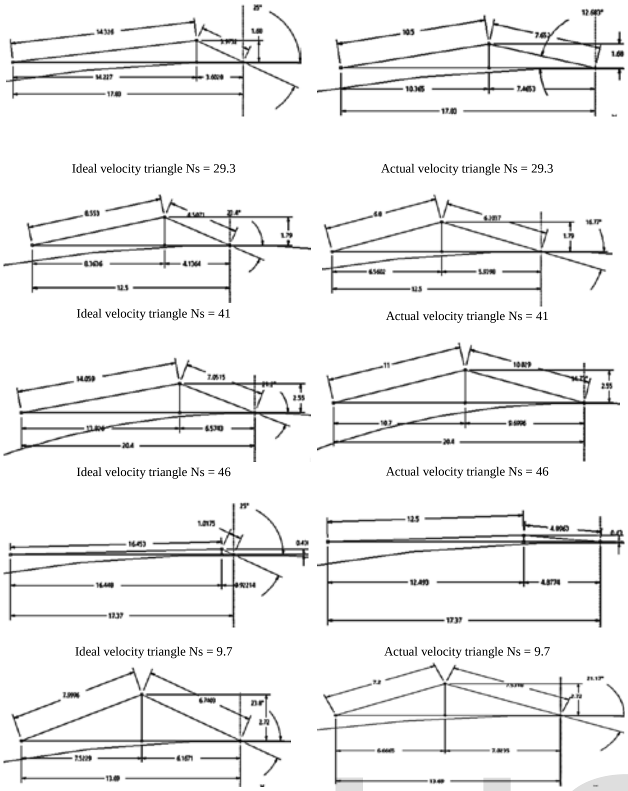
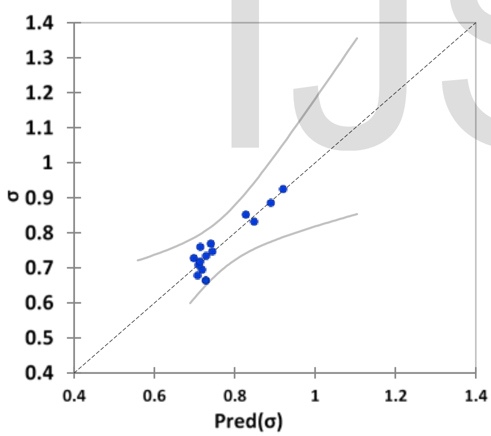


Fig 13 Ideal and actual velocity triangle at exit of impeller



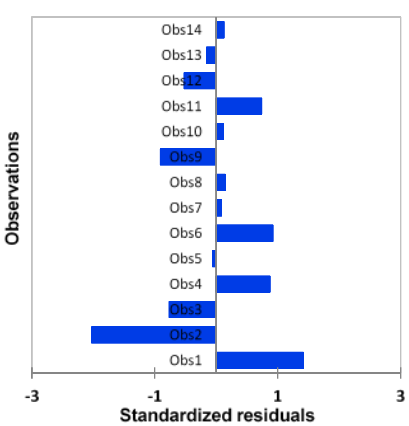
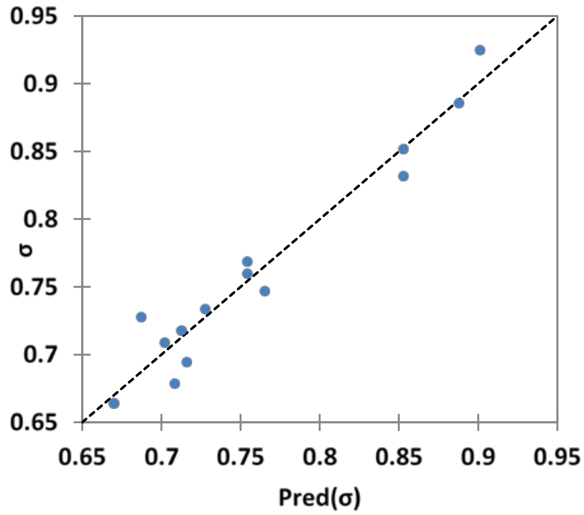


Fig 14 Present and predicted slip factor residual for equation 6



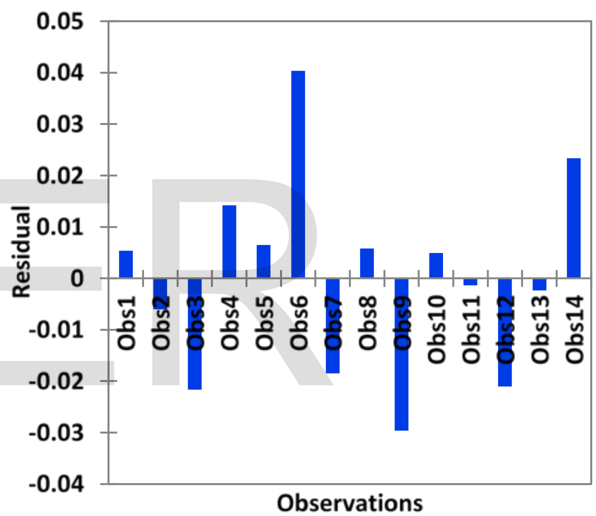


Fig 15 Present and predicted slip factor residual for equation 7

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Nomenclature

σ	Slip factor	Ns	Specific speed
δ	Deviation angle	Q	The pump discharge m^3/hr
β_{2B}	The ideal exit vane (flow) angle	H	The pump head m
β_2	The actual exit vane (flow) angle	N	The pump speed RPM
Z	The number of impeller's vane	d_{sh}	The shaft diameter mm
$c_{2u\infty}$	The ideal whirl velocity	d_1	Suction diameter of impeller mm
c_{2u}	The actual whirl velocity	d_2	Impeller diameter mm
U	The impeller velocity	b_2	Impeller thickness mm
$W_{2\infty}$	The ideal flow velocity	d_3	Volute inlet diameter mm
W_2	The actual flow velocity	d_4	Volute out diameter mm
Δc_{2u}	Slip velocity	b_3	Volute thickness mm
C_{2m}'	Radial flow velocity	t_{blad}	Blade thickness mm
Φ	Impeller discharge flow coefficient, dimensionless C_{m2}/U	ρ	Density of fluid kg/m^3
σ_x	Normal stress in X-direction $Pascal/m^2$	τ_{xy}	Shear stress in XY plane $Pascal/m^2$